The Fractal world of Euler

Who was Leonhard Euler?

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Leonhard Euler (1707 - 1783), pastell painting by E. Handmann, 1753.

Leonhard Euler was one of the greatest mathematicians of all times. He developed the basics of the modern theory of numbers and algebra, the topology, the probability calculus and combinatorics, the integral calculus, the theory of the diffenrential equation and the differential geometry, the variational calculus and he discovered the coherence between trigonometrical functions and exponential functions. Leonard Euler developed the hydrodynamics and fluidic, he made the bases for the theory of the gyroscope. He was a brilliant natural scientist, an excellent teacher and mentor.

1851 the formerly two-storey residential house of Leonard Euler was upgraded. 1858 - 1917 girl secondary school 1918 - 1958 secondary modern school, today secondary modern school with specialized teaching curriculum in history, languages and literature. St. Petersburg, Lieutenant-Schmidt-Shore 15.

It was on April 24th, 1727 when on invitation of the Russian czarina Katharina I the 19-year-old master Leonhard Euler left his home town of Basel and set off for a brillant scientific career at the Academy of the Sciences of Russia. The brothers Bernoulli (Nikolai and Daniel), Christian Goldbach and other excellent European scientist already worked there. Peter I engaged the philosopher and mathematician Christian Wolf from Marburg shortly before his death to unite the best heads of Europe under the seal of Academy of the Sciences of St. Petersburg.

In May 1771 an enormous blaze raged through St. Petersburg. Hundreds of buildings burned down, among others the house of the graduate Leonard Euler. But the basler craftsman Peter Grimm succeeded in saving the 64-year-old blind mathematician from death by burning. Thanks to his courageous intervention almost all manuscripts of the greatest mathematician of all times remained for the posterity. Among others also the work “About continued fractions” (1737) and “About the vibrations of a string” (1748). In these papers Euler formulated theses whose solution would keep mathematics busy for 200 years to come. Eulers work made it possible 250 years later to air one of the most fundamental secrets of nature – the free vibrations of the universe.

Euler examined already free vibrations of an elastic thread with no mass occupied with pearls. In connection with this task d'Alembert developed his integration method for a system of linear differential equations. Starting out from there Daniel Bernoulli put forward his famous theorem that the solution for the problem of a free vibrating string can be portrayed as a trigonometrical series, which lead to a discussion between Euler, d'Alembert and D. Bernoulli which spread over few decades. Later on Langarne pointed out more correctly, how one can come to a solution of the problem of swinging string of beads to the solution of the problem of the vibrating of a homogeneous string by breaching the limit. Only in 1822 J. B. Fourier in solved this formulation completely for the first time.

Meanwhile, nearly insurmoutable problems still arose with pearls of various mass and irregular distribution. This task leads to functions with gaps. According to a letter of Charles Hermite of May 20th, 1893, which called to "reject the lamentable plague of the functions without derivations in fright and fear ", T. Stieltjes examined functions with discontinuities and found an integration method of such functions, which led to continued fractions.

The grave of Leonhard Euler in St. Petersburg.

Meanwhile, Euler already recognized that complicated vibrating systems can contain also such solutions (integrals) which aren't differentiating everywhere and left to the mathematically talented posterity an analytic monster – the so called non-analytic functions (this term was chosen by himself). Non-analytic functions have ensured a lot of work up until the 20th century, even after the identity crisis of the mathematics seemed to be already overcome.

The crisis started, as E. du Bois Reymond 1875 reported for the first time about a steady, but not subtly differentiable function designed by Weierstrass, and lasted approximately till 1925. Their dominant players were Cantor, Peano, Lebesgue and Hausdorff. As result a new branch of the mathematics was given a birth – **the fractal geometry**.

Fractal comes from the Latin fractus and means as much as "in pieces broken" and "irregular". So **fractal** are really incomplete, spiteful mathematical objects. The mathematics of the 19th century took these objects for exceptions and therefore looked at regular, steady and smooth structures or tried to put down **fractal** phenomenons to such structures.

This plaque was installed 1957 in honour of the 250th birthday of Leonard Euler at his former residential house at the shore of the Neva.

The theory of the **fractal** quantities made it possible to examine strictly “not analytic” creased, granulous or incomplete forms qualitatively. Soon it turnes out that **fractal** structures aren't that rare at all. In nature one discovered more **fractal** objects than suspected till now. More, it seemed so as if suddenly the universe was **fractal** by nature.

Especially the works of Mandelbrot placed the geometry finally in a position capable of describing correctly **fractal** mathematical objects: incomplete crystal lattices, the Brown’s movement of the gas molecules, sinuous polymer giant molecules, irregular star clusters, Cirrus clouds, the saturn rings, the distribution of the lunar craters, turbulences within liquids, bizarre shorelines, winding river courses, folded mountain ranges, branched forms of growth of most different plant sorts, areas of islands and seas, rock formations, geological depositions, the spatial distribution of raw material occurrences and so on and so on.

The Leningrad mathematicians F. R. Gantmacher and M. G. Krein looked 1950 at the deflection line of a vibrating string with pearls as partitioned line. Just this attempt made it possible for them to view the problem in a **fractal** way without being conscious of it (Mandelbrot’s classic “Le Objets **Fractals**” appeared 1975, his first works from 50’s fell into the linguistics school). Only the **fractal** view put them to the position to completely solve (also for the most general case) the 200 years old Euler’s problem of the vibrating string of beads for pearls of various masses and irregular distribution. In their work “Oscillation-Matrixes, Oscillation-Cores and Small Vibrations of Mechanical Systems” they proved, that all free vibrations form a finite string of beads or string a finite or infinite Stieltjes-continued fraction. The masses of the pearls and the separations between them are identical with the part denominators of the continued fraction.

1955 V. P. Terskich generalized the (as regards content **fractal**) continued fraction method on vibrations of complicated branched chain systems. The classic work of T. N. Thiele, A. A. Markov, A. J. Chintchin, O. Perron, J. A. Murphy, M. R. O'Donohoe, A. N. Chovansky, H. S. Wall, D. I. Bodnar, C. I. Kucminskaja, V. J. Skorobogat'ko and others helped to get the definite breakthrough for the continued fraction method and made the development of efficient algorithms possible for the addition and multiplication of continued fractions up till 1981.

Mathematical models of vibrating **fractal** chain systems are used with great success in the most different scientific fields today. Their popularity reached a highlight already in the sixties in the electrotechnical engineering. The fast development of the computer branch during the last decades have to be put down to the efficiency of such models in the solid state physics. One discovered vibrating **fractal** chain systems also in neural networks, our genome and eco-systems.

The complete universe is a vibrating **fractal** chain system, what can be compared mathematically with the free vibrations of a Euler’s string of beads of gigantic proportion. The natural oscillations of matter influence not only the temporal course of all physical, chemical, biological and social processes, but it is also a global morphogenetical factor and cause for a global selection process. Their frequency spectrum is logarithmic **fractal**.

Leonard Euler left about 900 scientific work.

Note:

for more information about fractals, especially 3D printed fractals and its fractal geometry, see:

In generally see: www.fractal.org and its search machine: www.fractal.org/search.asp

For Julius Ruis Set:

www.fractal.org/Julius-Ruis-set.pdf

For Fractal Growth on YouTube:

http://uk.youtube.com/julesruis

For the Julius Ruis 'Fractal Science and Art Gallery' see:

www.fractal.org/Julius-Ruis-Gallery/Index-Gallery.htm

For 'Fractalary: Fractals from Planet to Atoms' see (downloading will take about 30 seconds):

www.fractal.org/Fractalary/Fractalary.htm

For Fractal Tree Scaffold for Tissue Engineering

www.fractal.org/Fractal-tree-scaffold.htm

For making slices (.bmp files) of 3D Mandelbrot set for Rapid Prototyping

www.fractal.org/Fractal-Builder.exe

For some prototypes of Fractal Objects (e.g. blood vessels):

www.fractal.org/Prototype-Fractal-Geometry.pdf

For a trail version of the Fractal Imaginator:

http://www.mysticfractal.com/fractalimaginator_trial.htm

For article about Complex Adaptive and Emergent Systems (CAES) = Fractal Systems, see:

www.fractal.org/Fractal-systems.htm

And last but not least:

www.fractal.org/Julius-newtree-set.pdf

and

www.fractal.org/Mandelbulb.pdf

and 3D Juliabulbs animations:

www.youtube.com/julesruis?gl=GB&hl=en-GB

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